

New magnetic field induced macroscopic quantum phenomenon in a superconductor with gap nodes

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Abstract

High- T_c superconductivity is unconventional because the gap is not isotropic as in simple metals but has $d_{x^2-y^2}$ symmetry with lines of nodes. In a fascinating thermal transport experiment on a high- T_c superconductor, Krishana et al [1] have reported mysterious magnetic field induced first order transitions from a superconducting state with gap nodes to a state without gap nodes. We show here that this is an experimental manifestation of a novel *macroscopic* quantum phenomenon induced by the magnetic field, qualitatively different from the usual quantum Hall effects. It corresponds to the *quantization of the superfluid density* in a superconductor with gap nodes due to the generation of Confined Field Induced Density Waves (CFIDW) in the node regions of the Fermi surface. The Landau numbers L are not sufficient to index these macroscopic quantum states and the addition of a new quantum number ζ is necessary. Distinct qualitative implications of this non-integer $|L, \zeta >$ quantization are also evident in a number of recent unexplained experimental reports in the cuprates.

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In conventional superconductors the gap is rather isotropic having s-wave symmetry. A breakthrough in the study of high- T_c oxide superconductors (HTSC) was the demonstration that instead, the gap is very anisotropic of d -wave symmetry taking a zero value in the so called node directions [2, 3]. Many of the peculiar properties of HTSC are now simply understood as resulting from this distinct momentum structure of the gap. However, understanding the influence of a *perpendicular* to the $Cu - O$ planes magnetic field on this ground state is actually a fascinating challenge for theorists [4]. In a remarkable thermal transport experiment, Krishana et al. [1] have reported that, when a magnetic field is applied perpendicular to the CuO_2 planes, the thermal conductivity shows sharp first order transitions from a field dependent regime characteristic of gap nodes [5] to a field independent regime indicating the elimination of the nodes.

The results of [1] stimulated a controversial experimental and theoretical debate which was in fact never closed. Thermal transport measurements by other groups [6, 7, 8] have confirmed the surprising findings of Krishana et al.[1]. However, they have also reported that the phenomenon is not present in all samples [8]. Very high quality samples are apparently necessary [9]. In addition, strong hysteretic behavior was reported in Ref. [6] in disagreement with Refs. [1, 7]. Our theory provides a simple explanation of this controversy and the sample dependence of the occurrence of the phenomenon is a major argument for the relevance of our picture. On the theoretical side, the phase-transition-point of view has been immediately adopted by Laughlin who suggested that this is a transition from $d_{x^2-y^2}$ superconductivity (SC) to $d_{x^2-y^2} + id_{xy}$ SC [10] which is nodeless. Models based on the vortex dynamics have also been proposed [11] explaining the saturation of the field effect but could not account for the “kink” and the sharp character of the transition.

Indications of a field induced transition to a nodeless state are also present in other experiments. It has been suggested that the field induced splitting of the zero bias conductance peak in tunnel measurements may be the result of such a transition [12]. More spectacular are recent measurements by Sonier et al. [13] of the penetration depth in the presence of a stronger than usually magnetic field. They also confirm the elimination of the nodes by the magnetic field adding a fundamental new element: *The elimination of the nodes is accompanied by a substantial reduction of the superfluid density* [13]. Such a reduction cannot be explained by a phase transition to a new SC state like $d_{x^2-y^2} + id_{xy}$.

Moreover, a number of seemingly unrelated experimental puzzles emerged recently. Neutron scattering experiments reported the generation of AFM moments in the SC state by a magnetic field applied perpendicular to the planes [14]. The moments appear suddenly above a critical field and below a critical temperature [15]. NMR measurements have confirmed the phenomenon [16]. Scanning tunneling microscopy results report in the presence of a perpendicular magnetic field a checkerboard structure that covers a region around each vortex [17]. Finally, measuring both heat capacity and magnetization on an extra clean sample of $\text{YBa}_2\text{Cu}_3\text{O}_7$, Bouquet et al. [18] not only confirm the presence of a first order transition from a vortex lattice to a so called vortex glass state, but at higher fields they observe a surprising transition from the vortex glass state to a new vortex glass state which has not found any theoretical explanation so far.

In this Letter we explore an original physical picture which is shown to account for *all the experiments*. We reveal a new magnetic field induced phase transition from a $d_{x^2-y^2}$ SC state to a state in which $d_{x^2-y^2}$ SC *coexists with Confined Field Induced (Spin and Charge) Density Waves (CFIDW) which develop in the gap node regions of the Fermi surface (FS)*. Field induced density waves have been suggested [19, 20, 21, 22, 23] in

order to explain second order metal-insulator transitions in $(TMTSF)_2X$ ($X = PF_6, ClO_4$) quasi one-dimensional synthetic compounds under pressure [24]. The transitions we discuss here are novel and the qualitative physics of our CFIDW states is novel as well. We consider an hybrid approach. Our HTSC system is built of two subsystems: subsystem I is the Fermi Surface (FS) region covered by the superconducting gap and subsystem II is a virtual normal quasiparticle region created by the magnetic field and centered in the node points of the FS. The CFIDW will eventually develop in region II because of the orbital effect of the field in this region. The orbital effect of the field in region I induces vortices that are assumed to be irrelevant. The Zeeman effect is not considered here for simplicity, however in region II it can be shown to further stabilize our CFIDW states while in region I it is negligible. The relative momentum extension of regions I and II is unknown and fixed by the energetic competition of CFIDW with SC. The above picture has some similarity with the partially depaired Fulde-Ferrell-Larkin-Ovchinnikov state [25, 26] where the normal state is created by the magnetic field over a portion of the Fermi surface and competes with superconductivity.

In HTSC the gap is $d_{x^2-y^2}$ with nodes in the $(\pm\pi, \pm\pi)$ directions where region II is centered. In region II *we necessarily have open FS sheets*. Therefore, we can write the dispersion of subsystem II in the form: $\xi_{\mathbf{k}}^{II} = v_F(|k_1| - k_F) - 2t_2 \cos(k_2/X) - 2t'_2 \cos(2k_2/X)$ where X is the unknown momentum extension of region II (see Fig. 1a). k_1 is along the $(\pm\pi, \pm\pi)$ directions perpendicular to the open FS sheets of subsystem II, and k_2 is perpendicular to k_1 and therefore along the open FS sheets where we keep only two harmonics without influence on the generality of the results.

We are able to calculate explicitly the spin and charge susceptibility of subsystem II in the presence of the magnetic field exploiting methods developed for the study of

$(TMTSF)_2X$ compounds [20, 21]. The details of the calculation and a discussion with various electronic dispersions in II will be given elsewhere. We report here the simplest results sufficient to illustrate the surprising physics of the system. Constraining the nesting in the $(\pm\pi, \pm\pi)$ directions one can show that a first order field induced density wave gap in region II is given by $\Delta_{DW} = W \exp\{-[gN(E_F)I_L^2(X)]^{-1}\}$ where

$$I_L(X) = \sum_n J_{L-2n}\left(\frac{4t_2X}{eHv_F}\right) J_n\left(\frac{2t'_2X}{eHv_F}\right) \quad (1)$$

Here $J_n(x)$ are Bessel functions, L is the index of the Landau level configuration, e is the charge of the electron, H the magnetic field, $N(E_F)$ the density of states at the Fermi level (in region II), g a scattering amplitude of Coulombic or phononic origin, W the bandwidth in the (π, π) direction and v_F the Fermi velocity. Higher order gaps are not reported here for sake of clarity.

CFIDW states will develop only if the absolute free energy gain of the system due to the opening of a CFIDW gap in region II is bigger than that lost by the elimination of SC from this region. This condition implies the following inequality:

$$I_L^2\left(k_F \sin(Z\pi/2)\right) > \frac{2}{gN(E_F)} \left[\ln \frac{2W^2\pi Z}{\Delta_{sc}^2[\pi Z - \sin(\pi Z)]} \right]^{-1} \quad (2)$$

Z is a dimensionless number varying from 0 to 1 that represents the relative extension of the CFIDW states over the FS and is related to X by $X \approx k_F \sin(Z\pi/2)$. Δ_{sc} is the maximum value of the $d_{x^2-y^2}$ SC gap. The CFIDW state must be *confined in momentum space* with a DW gap smaller or equal to the absolute superconducting gap in the borders of region II. We therefore have:

$$I_L^2\left(k_F \sin(Z\pi/2)\right) \leq \left[gN(E_F) \ln \frac{W}{\Delta_{sc} \sin(Z\pi/2)} \right]^{-1} \quad (3)$$

The equality in (3) fixes the relative extension Z of the CFIDW (for each L) and

therefore $I_L^2 \left(k_F \sin(Z\pi/2) \right)$ which then fixes the CFIDW gap Δ_{DW} and the critical temperature T_{DW} at which the CFIDW forms. A graphic solution for Z is shown in Fig. 1b. Surprisingly, there are two possible values of Z for a given Landau level configuration. Therefore, the Landau numbers L are not sufficient to index our quantum states. A *new quantum number* ζ , associated with the two possible momentum extensions Z for each L configuration must be added. Physically ζ will index *the quantization of the superfluid density*. In fact, each quantized value of Z corresponds to a different relative extension of the SC region over the FS and therefore to a different density of superfluid carriers.

Using for our parameters values extracted from the experiments on HTSC and assuming a conventional scattering $gN(E_F) \approx 1$ we obtain results like those reported in Figures 2 and 3 in remarkable agreement with the data of [1] and [13]. In Fig. 2 is reported the dependence of T_{DW} on the magnetic field in the various $|L, \zeta >$ configurations. We also plot in this figure the experimental points of refs [1] and [13]. In Fig. 3 we plot the corresponding magnetic field dependence of the accessible relative extensions Z of the CFIDW in each $|L, \zeta >$ configuration for the same parameters. A quantitative fit of the experiments as in figure 2 establishes that the orders of magnitude of the involved parameters are compatible with the experimental data. More importantly, there are distinct qualitative experimental facts which strongly support our picture. As one can see in Fig. 2, the T_{DW} versus critical magnetic field profile of the data in [1] (circles) show a “*reptation*” shape. The first two points have a bigger field slope than the next two points and so on. To the best of our knowledge, no explanation of the reptation behavior has been reported so far. Within our analysis this “reptation” profile is due to the *quantization* of the CFIDW states. Each slope corresponds to a different $|L, \zeta >$ quantum configuration of the system. The higher the field is, the smaller is the field slope in agreement with the

experiment. Moreover, if one associates the vortex solid to vortex glass transition with the SC to SC+CFIDW transition, one naturally understands *the unexplained vortex glass I to vortex glass II transition of Bouquet et al [18] as a transition between two different $|L, \zeta >$ configurations of the SC+CFIDW state.* The higher field $|L, \zeta >$ state corresponds to a higher relative extension of the CFIDW state and therefore a less rigid vortex structure as is experimentally observed. Altogether, this provides a microscopic viewpoint for the origin of the spin-glass states.

The remarkable sensitivity of the occurrence of this phenomenon on sample quality [8, 9] further corroborates our picture. Within our analysis, a small magnetic field affects the large SC gap of HTSC creating CFIDW's in the node region *because of momentum confinement* which constrains the cyclotron orbits to be large in real space and consequently the involved flux is large as well. The occurrence of the CFIDW's *requires very clean samples* because the mean free path must be bigger than the cyclotron orbits. Samples which may appear of high quality from the usual criteria (width of the SC transition or magnitude of the SC T_c) may not be sufficiently clean to support large cyclotron orbits and show the CFIDW states. This explains the puzzling sample dependence of the occurrence of the phenomenon. The minimum field at which the phenomenon is observable is also limited by sample defects because if fields are too small, momentum extensions of the CFIDW are also too small (see figure 3) and consequently the required cyclotron orbits are too large. Our $d_{x^2-y^2}$ to $d_{x^2-y^2} + CFIDW$ transition appears only above 1 Tesla in [6] while it is already present at 0.6 Tesla in [1] because samples in [1] are cleaner admitting bigger cyclotron orbits. Because all $|L, \zeta >$ configurations are nearly degenerate for the total system, the occurrence of magnetic hysteresis on a dynamic probe like thermal transport will depend on the exact conditions of the magnetic cycle and on sample

quality as well. In the magnetic cycle of Ref. [6] showing hysteresis, the field orientation is reversed when the field maxima are reached (the field passes through zero), while in Refs. [1, 7] this is not the case and hysteresis is absent.

Our analysis is also the first to establish a natural relationship between the thermal transport [1] and penetration depth [13] puzzles. In [13] CFIDW develop at a much higher T_{DW} for a given field than in [1] because the system is cooled in the presence of the magnetic field. By field cooling the sample, the first accessible $|L, \zeta >$ configuration is the one with the higher T_{DW} which also corresponds to the higher Z (cf. figure 3). On the other hand, in the experiment of [1], the temperature is kept constant when the field varies. Not only we reproduce simultaneously the experimental T_{DW} versus field profiles of both [1] and [13], but we also account for the reduction of the superfluid density reported in [13]. As one can see in figure 3, in the regime 4 - 6 T explored in [13], the CFIDW occupy about a quarter of the FS ($Z \approx 0.25$ to 0.35) which is in very good agreement with the $\approx 15\%$ to 25% reduction of the superfluid density reported in [13]. Furthermore, our CFIDW states correspond to a real space pattern that has all the characteristics of the one observed by STM [17].

Finally, in agreement with all the experiments, our CFIDW states appear *only for fields perpendicular to the planes* because for fields parallel to the planes cyclotron effects are irrelevant.

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Figure Captions

Figure 1: a): Schematic view of the SC-CFIDW competition. b): Graphic solution of (3) in the $L = 1$ configuration and a field of 3 Tesla. Two different relative extensions Z ($X \approx k_F \sin(Z\pi/2)$) of the CFIDW are possible.

Figure 2: (color): Critical temperature T_{DW} versus critical field for the formation of a CFIDW state in the different quantum configurations $|L, \zeta \rangle$: $L = 4$ (orange), $L = 3$ (red), $L = 2$ (blue), and $L = 1$ (green). In all L configurations, full lines and dotted lines correspond to the two different ζ configurations (full lines to the higher Z solution). The $L = 0$ lines are not shown for clarity. The open circles are the corresponding experimental points of [1] and the open squares the experimental points of [13].

Figure 3: (color): Relative extensions Z versus magnetic field for various quantum configurations $|L, \zeta \rangle$. The different colors and line-styles correspond to the same $|L, \zeta \rangle$ as in Fig. 2.







